

Flux exchange in inhomogeneous type-II superconductors

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The vortex hopping motion in a type-II superconductor determines the current-carrying ability and consequently the application fields of the superconductor. However, it is not clear how the vortices hop between the different pinning regions in the superconductor. Here we proposed that there should be magnetic *flux exchange* between two contacting pinning regions. A system of differential equations was constructed to describe the flux exchange phenomenon. The numerical solutions and approximate solutions of the system were obtained. The results show that the flux exchange reduces the internal field in the weak pinning regions, but increases the internal field in the strong pinning regions. Moreover, the flux exchange phenomenon is strongly influenced by the superconductor's geometrical size.

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I. INTRODUCTION

The vortices in a type-II superconductor are generally pinned down by pinning centers.¹⁻³ But there is a probability for the vortices to spontaneously hop between the adjacent pinning centers due to the vortex thermal fluctuation.¹⁻⁷ The hopping motion can be described by the Arrhenius equation, which shows that the hopping frequency is strongly related to the activation energy of the vortices.⁷ The hopping motion causes the magnetic flux (or current) in the superconductor to reduce with increasing time, which is usually referred as flux relaxation. This phenomenon exhibits various time evolution behaviors because of the complicated field (or current) dependence of the activation energy.⁷⁻¹⁵ Due to the dissipation of the hopping motion, it determines the superconductor's current carrying ability and consequently its application fields.^{1,2,16,17} Thus, a large number of experimental and theoretical research works^{1,3,16-24} have been carried out to study the vortex hopping motion.

In the present work, we wish to study the mutual vortex hopping motion between the contacting regions with different pinning ability, which we shall call *flux exchange*. More specifically, a real superconductor may be divided into smaller regions whose sizes are larger than the superconductor's penetration depth. Because of the different pinning ability of these smaller regions,²⁵ the vortices close to the interface between two contacting regions will hop from one region into another region²⁶⁻²⁸, and vice versa. In other words, the two contacting pinning regions exchange vortices (flux). Because of the different pinning ability, the vortex hopping motions in each smaller region need to be described by different equations. This indicates that a flux exchange process is determined by the parameters of both contacting pinning regions. It can not be simply described by the conventional flux relaxation theory. Thus, we need to consider the *flux exchange* seriously and construct new mathematical equations to describe it.

However, the flux exchange is usually obscured by the dominant flux relaxation and is then ignored in magneti-

zation (or internal field) measurements carried out over the entire sample.^{1,2,29,30} To study the flux exchange phenomenon, we may consider the fact that the vortices in a weak pinning region have a higher energy and higher hopping frequency, but the vortices in a strong pinning region have a lower energy and lower hopping frequency. Consequently, there should be more vortices hop from the weak pinning region into the strong pinning region in a unit time interval. It will result in a nonzero vortex migration from the weak pinning region to the strong pinning region, which changes the vortices quantity and consequently the flux relaxation law in each pinning region. This indicates that it is possible to study the flux exchange by investigating its influences on the flux relaxation behavior.

In this work, the physical principle of the flux exchange phenomenon was analyzed. First, we constructed a system of differential equations to describe the flux exchange phenomenon by investigating its influence on the flux relaxation behavior. Next, we calculated the numerical results of the system to view the influence of the flux exchange on the flux relaxation. Finally, we calculated the analytical solutions of the system under a special approximation. The flux exchange between the bulk pinning region and surface pinning region of a type-II superconductor was also discussed.

II. DIFFERENTIAL EQUATIONS OF FLUX EXCHANGE

As mentioned before, the flux exchange here means the mutual vortex hopping motion between different pinning regions. In this section, we shall first consider a flux exchange process that only involves two pinning regions (with sizes larger than the superconductor's penetration depth): the p -th pinning regions and q -th pinning regions (see Fig. 1). Later, we shall generalize it into a flux exchange process that involves N different pinning regions.

To describe the flux exchange phenomenon, we need to construct a system of differential equations. This can be

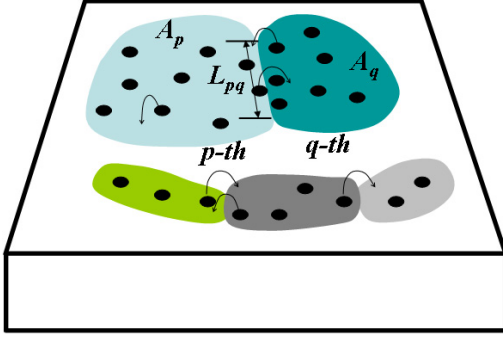


FIG. 1: (Color online) Schematic diagram of magnetic flux exchange between the p -th pinning region and q -th pinning region of a type-II superconductor. The black disks represent vortices. The arrows represent the hopping directions of the vortices. A_p is the cross area perpendicular to the magnetic field in the p -th pinning region. A_q is the cross area perpendicular to the magnetic field in the q -th pinning region. L_{pq} is the line length on the interface perpendicular to magnetic field.

achieved by considering the rate of change of the magnetic flux in each pinning region, which is proportional to vortex hopping frequency.^{7,14,31} The hopping process can be described by the Arrhenius equation $\nu = \nu_0 e^{-U/kT}$, where ν is hopping frequency, ν_0 is attempt frequency, U is vortex activation energy, k is Boltzmann constant, and T is temperature. Multiplying ν by the average hopping distance s , we obtain the vortex velocity^{8,32}

$$v = s\nu_0 e^{-U/kT}. \quad (1)$$

In the p -th pinning region, the magnetic flux variation in a time interval Δt includes three parts: (a) Due to flux exchange, the vortices in the p -th pinning region (close to the interface) hop into the q -th pinning region^{8,32} by $-\Delta\Phi_{pq} = -\frac{1}{2}B_p L_{pq} v_p \Delta t$, where B_p is average internal field (vortex density) in the p -th pinning region, L_{pq} is the line length on the interface perpendicular to field, v_p is the vortex velocity in the p -th pinning region (see Eq.(1)). The number $\frac{1}{2}$ accounts for the random hopping motion

in both directions. (b) Due to flux exchange, the vortices in the q -th pinning region hop back to the p -th pinning region by $\Delta\Phi_{qp} = \frac{1}{2}B_q L_{qp} v_q \Delta t$, where B_q is average internal field in the q -th pinning region, $L_{qp} = L_{pq}$, and v_q is the vortex velocity in the q -th pinning region. (c) Due to flux relaxation (dissipative), the vortices in the p -th pinning region decrease by $-\Delta\Phi_{pr} = -\epsilon_p A_p e^{-U_p/kT} \Delta t$, where ϵ_p is a proportional constant³¹ and A_p is the cross area perpendicular to the magnetic field in the p -th pinning region.

The total change of magnetic flux in the p -th pinning region is now $\Delta\Phi_p = -\Delta\Phi_{pr} - \Delta\Phi_{pq} + \Delta\Phi_{qp}$. Dividing both sides by $A_p \Delta t$, we obtain a differential equation for the average internal field in the p -th pinning region, that is,

$$\frac{dB_p}{dt} = -\epsilon_p e^{-U_p/kT} - \epsilon_{pq} B_{pq}, \quad (2)$$

where $\epsilon_{pq} = \frac{1}{2}L_{pq}s\nu_0/A_p$ and

$$B_{pq} = B_p e^{-U_p/kT} - B_q e^{-U_q/kT}. \quad (3)$$

is a function related the flux exchange (Eq.(1) is used).

In the q -th pinning region, similarly, the total change of magnetic flux is $\Delta\Phi_q = -\Delta\Phi_{qr} - \Delta\Phi_{qp} + \Delta\Phi_{pq}$, where $-\Delta\Phi_{qr} = -\epsilon_q A_q e^{-U_q/kT} \Delta t$, ϵ_q is a proportional constant, and A_q is the cross area perpendicular to the magnetic field in the q -th pinning region. Dividing both sides by $A_q \Delta t$, we obtain a differential equation for the average internal field in the q -th pinning region, that is,

$$\frac{dB_q}{dt} = -\epsilon_q e^{-U_q/kT} - \epsilon_{qp} B_{qp}, \quad (4)$$

where $\epsilon_{qp} = \frac{1}{2}L_{qp}s\nu_0/A_q$ and $B_{qp} = -B_{pq}$.

Eq.(2) and Eq.(4) constitute a nonlinear system of differential equations that describes the flux exchange between the p -th pinning region and q -th pinning region. If the flux exchange process includes N different pinning regions, then the system of differential equations should be generalized into

$$\left\{ \begin{array}{l} \frac{dB_1}{dt} = -\epsilon_1 e^{-U_1/kT} - \epsilon_{12} B_{12} - \dots - \epsilon_{1i} B_{1i} - \dots - \epsilon_{1N} B_{1N}, \\ \frac{dB_2}{dt} = -\epsilon_{21} B_{21} - \epsilon_2 e^{-U_2/kT} - \dots - \epsilon_{2i} B_{2i} - \dots - \epsilon_{2N} B_{2N}, \\ \dots\dots\dots \\ \frac{dB_i}{dt} = -\epsilon_{i1} B_{i1} - \epsilon_{i2} B_{i2} - \dots - \epsilon_i e^{-U_i/kT} - \dots - \epsilon_{iN} B_{iN}, \\ \dots\dots\dots \\ \frac{dB_N}{dt} = -\epsilon_{N1} B_{N1} - \epsilon_{N2} B_{N2} - \dots - \epsilon_{Ni} B_{Ni} - \dots - \epsilon_N e^{-U_N/kT}. \end{array} \right. \quad (5)$$

For simplicity, we can simply write system (5) as:

$$\frac{dB_i}{dt} = -\epsilon_i e^{-U_i/kT} - \sum_{j'=1}^N \epsilon_{ij} B_{ij},$$

where ϵ_i is a proportional constant and $\epsilon_{ij} = 1/2 L_{ij} s \nu_0 / A_i$, $B_{ij} = B_i e^{-U_i/kT} - B_j e^{-U_j/kT}$, and $i = 1, \dots, N$. The ' on j means $j \neq i$, i.e., the summation does not include ϵ_{ii} terms. If the i -th pinning region and j -th pinning region are not contacted, then their interface length $L_{ij} = L_{ji} = 0$. Consequently, $\epsilon_{ij} = \epsilon_{ji} = 0$ and the corresponding exchange terms vanish automatically (Note that generally $\epsilon_{ij} \neq \epsilon_{ji}$ unless $A_i = A_j$).

From mathematics we know that there is no general method available for calculating the exact solutions of a nonlinear system.³³ Thus, we shall analyze the nonlinear system using numerical methods and approximate methods. For simplicity, we shall discuss a flux exchange process that only involves two pinning regions Eq.(2), Eq.(4).

III. NUMERICAL SOLUTION

To carry out a numerical calculation of Eq.(2), Eq.(4), we need the initial conditions $B_p(0) = B_{p0}$, $B_q(0) = B_{q0}$ respectively. Let us now rewrite the system as

$$\begin{cases} \frac{dB_p}{dt} = -\epsilon_p e^{-U_p/kT} - \epsilon_{pq} (B_p e^{-U_p/kT} - B_q e^{-U_q/kT}), \\ \frac{dB_q}{dt} = -\epsilon_q e^{-U_q/kT} + \epsilon_{qp} (B_p e^{-U_p/kT} - B_q e^{-U_q/kT}), \\ B_p(0) = B_{p0}, B_q(0) = B_{q0}. \end{cases} \quad (6)$$

Furthermore, we still need the detailed expressions of the activation energies U_p and U_q . A number of activation energies were proposed in early studies.⁷⁻¹⁵ In principle, we can choose any one of these activation energies for both U_p and U_q . If the number of activation energies is N , then the number of systems of differential equations is N^2 . This is a big number and it is impossible to exhaust all the possible combinations in the present paper. To obtain a direct view on the effects of flux exchange, we shall do numerical calculations using the following two combinations:

- (a). Linear activation energy⁷: $U_p(B_p) = U_{p0}(1 - B_p/B_{p0})$, $U_q(B_q) = U_{q0}(1 - B_q/B_{q0})$.
- (b). Logarithmic activation energy¹³: $U_p(B_p) = U_{p0} \ln(B_{p0}/B_p)$, $U_q(B_q) = U_{q0} \ln(B_{q0}/B_q)$.

The parameters were put as follows: $B_{p0} = B_{q0} = 20$ G, $U_{p0}/kT = 15$, $U_{q0}/kT = 75$, $\epsilon_p = \epsilon_q = 0.6$ G/s, and $\epsilon_{pq} = \epsilon_{qp} = 0, 0.05$, and 0.50 s⁻¹ respectively. In the numerical calculations, we also noticed that the step of t must be very small. Otherwise, the results are unstable. It indicates that system (6) is a stiff system. The calculated results are shown in Fig 2.

Fig. 2 shows that the flux exchange reduces $B_p(t)$ (the internal field in p -th pinning region), but increases $B_q(t)$ (the internal field in the q -th pinning region). It looks that the q -th pinning region attracts vortices from the p -th pinning region. In fact, there is no attracting force in the q -th pinning region. It is just because we have assumed that the activation energy $U_p < U_q$, and therefore, the vortex hopping frequency $\nu_p > \nu_q$. This difference results in a “deficit” of flux exchange in the p -th pinning region, but a “surplus” of flux exchange in the q -th pinning region.

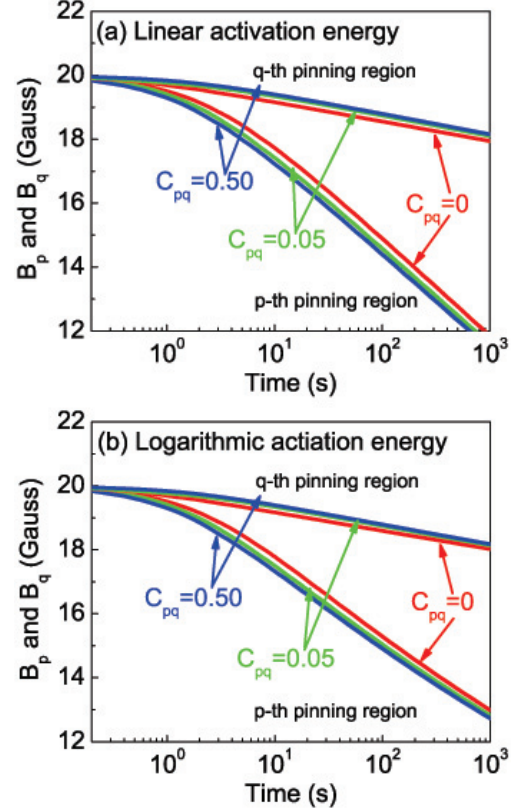


FIG. 2: (Color online) Numerical results of magnetic flux exchange. In the calculation, we used the following two activation energy combinations for the p -th and q -th pinning regions: (a) Linear activation energies $U_p(B_p) = U_{p0}(1 - B_p/B_{p0})$ and $U_q(B_q) = U_{q0}(1 - B_q/B_{q0})$. (b) Logarithmic activation energy $U_p(B_p) = U_{p0} \ln(B_{p0}/B_p)$ and $U_q(B_q) = U_{q0} \ln(B_{q0}/B_q)$. The parameters were put as: $B_{p0} = B_{q0} = 20$ G, $U_{p0}/kT = 15$, $U_{q0}/kT = 75$, $\epsilon_p = \epsilon_q = 0.6$ G/s, and $\epsilon_{pq} = \epsilon_{qp} = 0, 0.05$, and 0.50 s⁻¹ respectively.

IV. APPROXIMATE ANALYTICAL SOLUTIONS

In this section, we shall explore the analytical solutions of Eq.(2), Eq.(4) under the following approximation: *the effect of flux exchange is ignorable for the average internal field in the p -th pinning region $B_p(t)$, but is significant*

for the average internal field in the q -th pinning region $B_q(t)$. Using mathematical language, we can write out the approximation as: (a) $\epsilon_{pq}B_{pq} \approx 0$; (b) $|\epsilon_{qp}B_{qp}| \gg 0$.

Later, we shall show that the above approximation can be used to describe the flux exchange process between the bulk pinning region and surface pinning region of a type-II superconductor.

A. Solutions in the p -th pinning region

The first approximation, (a) $\epsilon_{pq}B_{pq} \approx 0$, indicates that $\epsilon_{pq} \approx 0$ (Note that $B_{pq} \neq 0$. Otherwise, $B_{qp} = -B_{pq} = 0$ and the second approximation (b) $|\epsilon_{qp}B_{qp}| \gg 0$ cannot be satisfied). This occurs when the area of the p -th pinning region A_p is so large that $\epsilon_{pq} = 1/2 L_{pq} s \nu_0 / A_p \approx 0$. Therefore, Eq.(2) reduces to

$$\frac{dB_p}{dt} \approx -\epsilon_p e^{-U_p/kT}. \quad (7)$$

Eq.(7) is a decoupled differential equation. It has exact solutions with the following two activation energies³⁴ (with initial condition $B_p(0) = B_{p0}$):

(1). Linear activation energy: $U_p(B_p) = U_{p0}(1 - B_p/B_{p0})$. The solution is

$$B_p(t) = B_{p0} \left[1 - \frac{1}{\beta_p} \ln \left(1 + \frac{t}{\tau_p} \right) \right], \quad (8)$$

where $\beta_p = U_{p0}/kT$ and $\tau_p = B_{p0}/(\beta_p \epsilon_p)$.

(2). Logarithmic activation energy: $U_p(B_p) = U_{p0} \ln(B_{p0}/B_p)$. The solution is

$$B_p(t) = B_{p0} \left(1 + \frac{t}{\tau_p} \right)^{-\gamma}, \quad (9)$$

where $\gamma = 1/(\beta_p - 1)$ and $\tau_p = B_{p0}\gamma/\epsilon_p$.

B. Solutions in the q -th pinning region

The second approximation, (b) $|\epsilon_{qp}B_{qp}| \gg 0$, indicates that $|B_{qp}| = |B_q e^{-U_q/kT} - B_p e^{-U_p/kT}| \gg 0$. This occurs in two cases:

(1). $B_q e^{-U_q/kT} \gg B_p e^{-U_p/kT}$, then $B_{qp} \approx B_q e^{-U_q/kT}$;
(2). $B_q e^{-U_q/kT} \ll B_p e^{-U_p/kT}$, then $B_{qp} \approx -B_p e^{-U_p/kT}$.

Here we shall calculate the solutions only using the linear activation energy⁷

$$U_q(B_q) = U_{q0}(1 - B_q/B_{q0}). \quad (10)$$

The reason is that the calculation is very difficult using other activation energies. Later we shall further show that Eq.(10) can be used to describe the vortex motion in surface pinning region.

$$1. \quad B_{qp} \approx B_q e^{-U_q/kT}$$

Substituting $B_{qp} \approx B_q e^{-U_q/kT}$ into Eq.(4), we have

$$\frac{dB_q}{dt} = -\epsilon_q e^{-U_q/kT} - \epsilon_{qp} B_q e^{-U_q/kT}. \quad (11)$$

Substituting Eq.(10) into Eq.(11), we have

$$\frac{e^{-x}}{x} \frac{dx}{dt} = -\epsilon_{qp} e^{-x_0} \quad (12)$$

where

$$x = \tilde{\beta}_q B_q + (\tau_q \epsilon_{qp})^{-1}, \quad (13)$$

$$x_0 = x(0) = \beta_q + (\tau_q \epsilon_{qp})^{-1}, \quad (14)$$

$\beta_q = U_{q0}/kT$, $\tilde{\beta}_q = \beta_q/B_{q0}$, and $\tau_q = 1/(\tilde{\beta}_q \epsilon_q)$.

Integrating both sides of Eq.(12) with respect to t , we have

$$\ln|x| + \sum_{n=1}^{\infty} \frac{(-x)^n}{n \cdot n!} = -(\epsilon_{qp} e^{-x_0}) t + C_1 \quad (15)$$

where the formula

$$\int \frac{e^{-x}}{x} dx = \ln|x| + \sum_{n=1}^{\infty} \frac{(-x)^n}{n \cdot n!},$$

is used and

$$C_1 = \ln|x_0| + \sum_{n=1}^{\infty} \frac{(-x_0)^n}{n \cdot n!}$$

is an integrating constant that is determined by the initial condition $x(0) = \beta_q + (\tau_q \epsilon_{qp})^{-1}$ (or $B_q(0) = B_{q0}$).

The internal field in the q -th pinning region, $B_q(t)$, is then determined by Eq.(13), Eq.(14), and Eq.(15).

$$2. \quad B_{qp} \approx -B_p e^{-U_p/kT}$$

Substituting $B_{qp} \approx -B_p e^{-U_p/kT}$ into Eq.(4), we have

$$\frac{dB_q}{dt} = -\epsilon_q e^{-U_q/kT} + \epsilon_{qp} B_p e^{-U_p/kT}. \quad (16)$$

Substituting Eq.(10) into Eq.(16), we have

$$\frac{dy}{dt} + \left(\epsilon_{qp} \tilde{\beta}_q B_p e^{-U_p/kT} \right) y = \frac{1}{\tau_q}, \quad (17)$$

where

$$y = e^{\beta_q(1-B_q/B_{q0})} = e^{\beta_q - \tilde{\beta}_q B_q}. \quad (18)$$

and the initial condition $B_q(0) = B_{q0}$ is now replaced by

$y(0) = 1$.

Rewriting Eq.(7) as $e^{-U_p/kT} = -(dB_p/dt)/\epsilon_p$ and substituting it into Eq.(17), we have

$$\frac{dy}{dt} + \left[-\frac{\epsilon_{qp}\tilde{\beta}_q}{2\epsilon_p} \frac{d}{dt} (B_p^2) \right] y = \frac{1}{\tau_q}. \quad (19)$$

Eq.(19) is a first order linear differential equation. Its general solution is

$$y(t) = P^{-1}(t) \left[\frac{1}{\tau_q} \int P(t) dt + C_2 \right], \quad (20)$$

where

$$P(t) = \exp \left[-\frac{\epsilon_{qp}\tilde{\beta}_q}{2\epsilon_p} B_p^2(t) \right] \quad (21)$$

and the C_2 in Eq.(20) is an integrating constant that is determined by the initial condition $y(0) = 1$. The $B_p(t)$ in Eq.(21) is the internal field in the p -th pinning region. It has two options, i.e., Eq.(8) or Eq.(9).

Finally, the internal field in the q -th pinning region can be obtained using Eq.(18), that is,

$$B_q(t) = B_{q0} \left\{ 1 - \frac{1}{\beta_q} \ln [y(t)] \right\}. \quad (22)$$

In the absence of flux exchange, the parameter $\epsilon_{qp} = 0$ (see Eq.(4)). Therefore, Eq.(22) reduces to a logarithmic function $B_q(t) = B_{q0} [1 - \beta_q^{-1} \ln (1 + t/\tau_q)]$. This is consistent with our assumption that the vortex activation energy in the q -th pinning region obeys the linear law $U_q(B_q) = U_{q0}(1 - B_q/B_{q0})$.

V. DISCUSSION

In this section, we shall prove that the system (Eq.(2), Eq.(4)) is equivalent to Maxwell's equations. We shall also discuss the possible flux exchange between the bulk pinning region and surface pinning region of a type-II superconductor.

A. Equivalent with Maxwell's equation

Because the vortices in a type-II superconductor are the quanta of magnetic field, their motions must obey the Maxwell's equation³², or continuity equation⁸. This equation gives

$$\partial_t B = -\partial_x (Bv). \quad (23)$$

where B is average internal field (vortex density) and v is the vortex velocity (see Eq.(1)). Let us now prove that Eq.(2), Eq.(4) are equivalent to Eq.(23).

As mentioned before, the vortex hopping motions in the smaller regions are described by different equations. This indicates that the right side of Eq.(23) cannot be directly used to describe the flux exchange between two contacting pinning region. To solve the problem, we may refer to the definition of one-sided derivatives in mathematics. Let us now consider the left derivative of Bv , that is,

$$\partial_- (Bv) = \lim_{h \rightarrow 0^-} \frac{B(x+h)v(x+h) - B(x)v(x)}{h}. \quad (24)$$

Let $x+h$ be a point in the p -th pinning region and x be a point in the q -th pinning region. Therefore, Eq.(23) can be rewritten as

$$\left(\frac{dB_p}{dt} \right)_{exchange} = -C_p (B_p v_p - B_q v_q). \quad (25)$$

where C_p is an unknown constant.

On the other hand, the dissipation of the vortex motion (flux relaxation) should also cause a reduction to B_p , the average internal field in the p -th pinning region. This process is usually described by an approximation³¹ of Eq.(23), that is,

$$\left(\frac{dB_p}{dt} \right)_{relaxation} \approx -\epsilon_p e^{-U_p/kT}. \quad (26)$$

Combing Eq.(25) and Eq.(26), we have

$$\frac{dB_p}{dt} = -\epsilon_p e^{-U_p/kT} - C_p s \nu_0 B_{pq}. \quad (27)$$

One can see that Eq.(2) and Eq.(27) are the same equation (Eq.(1) is used), where $C_p = 1/2 L_{pq}/A_p$. This shows that Eq.(2) is equivalent to the Maxwell's equation Eq.(23). Similarly, we can also prove that Eq.(4) is equivalent to Eq.(23).

B. Flux exchange between bulk pinning region and surface pinning region

In the superconductor's bulk, the vortices are subjected to weak random pinning^{9,10,35} and have a higher energy¹. In a thin layer close to the superconductor's surface, however, the vortices are subjected to strong surface pinning^{8,36-39} and have a lower energy¹. Therefore, the bulk pinning region and surface pinning region should exchange magnetic flux (see Fig. 3).

Let us now consider a cylindrical superconductor with a radius R . An external magnetic field is applied parallel to the axis of the superconductor. Assign the bulk pinning region as the p -th pinning region and surface pinning region as the q -th pinning region. Let δ be the thickness of the surface pinning region (δ is of the order of the superconductor's penetration depth, and $\delta \ll R$). Therefore, the area of the bulk pinning region

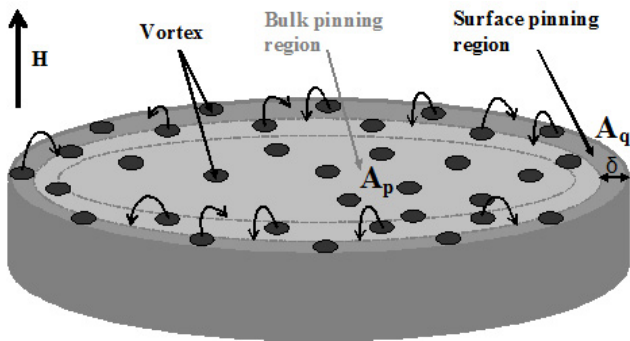


FIG. 3: (Color online) Schematic diagram of magnetic flux exchange between the bulk pinning region and surface pinning region of a type-II superconductor. The black disks represent vortices. The arrows represent the vortices' hopping directions. A_p is the cross area perpendicular to the magnetic field in the bulk pinning region. A_q is the cross area perpendicular to the magnetic field in the surface pinning region. δ is the thickness of the surface pinning region.

is $A_p = \pi(R - \delta)^2$, the area of the surface pinning region is $A_q = \pi R^2 - \pi(R - \delta)^2$, and the length of the line on the interface perpendicular to field is now the perimeter of the bulk pinning region, i.e., $L_{pq} = 2\pi(R - \delta)$.

Let $R \rightarrow \infty$, we have $L_{pq}/A_p = 2/(R - \delta) \rightarrow 0$, and consequently, $\epsilon_{pq} \approx 0$. This shows that the first approximation $\epsilon_{pq}B_{pq} \approx 0$ is satisfied. Thus, Eq.(7) can describe the internal field in the bulk pinning region. On the other hand, the bulk pinning is a weak pinning^{9,10,35} and surface pinning is a strong pinning^{8,36-38}, we have $U_p \ll U_q$, $B_p e^{-U_p/kT} \gg B_q e^{-U_q/kT}$, and consequently, $|B_{qp}| \approx |B_p e^{-U_p/kT}| \gg 0$. This shows that the second approximation $|\epsilon_{qp}B_{qp}| \gg 0$ is also satisfied. Thus, Eq.(16) can describe the internal field in the surface pinning region $B_q(t)$.

2. *Activation energy.*— In the bulk pinning region, there is a number of options for the activation energy $U_p(B_p)$. However, the exact solutions can be found only with the linear activation energy and logarithmic activation energy.³⁴

In the surface pinning region, let us now prove that the linear activation energy (see Eq.(10)) is more accurate than others. Recall that the vortex deformation and the nonlinear interaction between vortices result in the higher order (nonlinear) terms in the activation energy.¹⁴ However, the vortex deformation in the surface pinning region is small because the vortices are parallel to each

other. Moreover, the surface pinning force is stronger than the interacting force between vortices because the surface pinning is a strong pinning. Thus, we can safely ignore the higher order terms in $U_q(B_q)$ and assume that it obeys the linear law Eq.(10).

3. *Total internal field.*— In an experiment, a measured internal field is usually the total average internal field over the entire superconductor. This field can be defined as

$$B_t = \frac{\Phi_p + \Phi_q}{A} = (1 - \alpha)B_p + \alpha B_q, \quad (28)$$

where $A = A_p + A_q = \pi R^2$ is the total area of the surface perpendicular to the magnetic field. $B_p = \Phi_p/A_p$ and $B_q = \Phi_q/A_q$ are the average internal fields in the bulk pinning region and surface pinning region, respectively. The geometrical factor (area ratio)

$$\alpha = \frac{A_q}{A} = 1 - \left(1 - \frac{\delta}{R}\right)^2$$

is the weight of the contribution from the magnetic flux in the surface pinning region. Let $R \rightarrow \infty$, we have $\alpha \rightarrow 0$. Thus, Eq.(28) reduces to $B_t = B_p$. This means that, in a large superconductor, the contribution to the total average internal field B_t from the magnetic flux in the surface pinning region αB_q is ignorable. This property is completely determined by geometry and is nothing related to physics.

VI. CONCLUSION

In a type-II superconductor, the different pinning regions exchange magnetic flux (vortices). This phenomenon causes the vortices in a weak pinning region to migrate to a strong pinning region. Thus, it influences the flux relaxation behavior in each pinning region. The flux exchange phenomenon also occurs between the bulk pinning region and surface pinning region of a type-II superconductor. But the vortices hop from the surface pinning region to the bulk pinning region has little influence on the average internal field in the bulk pinning region. In the calculation of total average internal field, the contribution from the surface pinning region is significant in a small superconductor, but is ignorable in a large superconductor.

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